Scaling of resonance frequency for strong injection-locked lasers

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It has been shown that strong optical injection locking can significantly enhance the resonance frequency of semiconductor lasers. In this Letter, we describe the trade-off between the maximum resonance frequency enhancement and the quality factor (Q) of the lossless laser cavity and show that the time–bandwidth product (product of photon lifetime and maximum resonance frequency) is equal to one half the square root of the external power injection ratio. The theoretical model agrees well with our experimental data. © 2007 Optical Society of America

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Optical injection locking (OIL) of semiconductor lasers has come under increasing interest. Under strong optical injection, the resonance frequency of semiconductor lasers can be greatly enhanced, overcoming the fundamental limit of relaxation oscillation in free-running lasers. Both theoretical predictions and experimental demonstrations have been reported [1–3]. It has been shown that the maximum resonance frequency enhancement is proportional to the square root of the injection ratio. However, how the resonance frequency scales with the laser cavity design has not been explored. Such a scaling law is important because it provides a guideline for optimizing the laser to achieve the maximum possible resonance frequency.

In this Letter, we derive a simple expression for the maximum resonance frequency enhancement ($\Delta\omega_{R,max}$) and utilize it as a figure of merit to compare injection-locked systems with a wide range of cavity lengths of slave lasers, from vertical-cavity surface-emitting lasers (VCSELs) to edge-emitting lasers (EELs) (Fig. 1). We show that $\Delta\omega_{R,max}$ is inversely proportional to the quality factor (Q) of the lossless laser cavity and develop a time–bandwidth product that defines a trade-off between cavity Q and resonance frequency enhancement.

A common injection-locking model solves a set of three lumped-element differential equations that include the slave laser's field magnitude, phase, and carrier density. The field amplitude and phase equations are [4,5]

$$\frac{dA(t)}{dt} = \frac{1}{2}g[N(t) - N_{th}]A(t) + \kappa A_{inj} \cos \phi(t), \quad (1a)$$

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2}g[N(t) - N_{th}] + \kappa \frac{A_{inj}}{A(t)} \sin \phi(t) - \Delta\omega, \quad (1b)$$

where $A(t)$ and $\phi(t)$ are the internal slave field amplitude and phase, respectively; g is the linearized gain; $N(t)$ is the carrier number; N_{th} is the threshold carrier number; κ is the coupling coefficient; A_{inj} is the magnitude of the injected field; α is the linewidth enhancement parameter; and $\Delta\omega$ is the detuning frequency. In the literature, the injection ratio, one of the primary parameters in injection locking, is typically defined as

$$R_{int} = \left(\frac{A_{inj}}{A_0}\right)^2 = \frac{P_{inj,int}}{P_0}, \quad (2)$$

where A_0 is the internal slave free-running field magnitude, $P_{inj,int}$ is the internal injected power, and P_0 is the internal slave free-running power. Note that A_{inj} and A_0 are the injected and free-running fields *inside* the slave laser cavity, which are not empirically measurable values. Here, we define an *external* injection ratio, based on experimentally measurable values:

$$R_{ext} = \frac{P_{inj,ext}}{P_{out}}, \quad (3)$$

where $P_{inj,ext}$ is the injected power incident on the slave facet and P_{out} is the output power of the free-running slave.

We can relate the internal and external injection ratios by defining the power reflectivity of the injection facet as r:

$$\frac{R_{int}}{R_{ext}} = \frac{(1-r)^2}{r}. \quad (4)$$

This relates the internal injection ratio commonly used in theoretical papers with the external injection

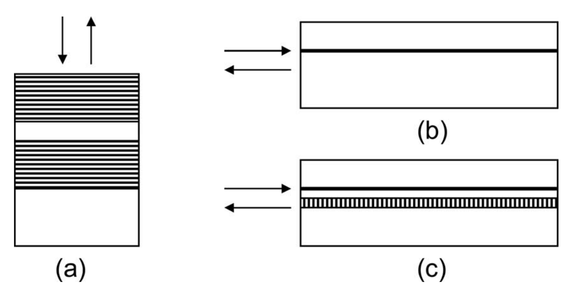

Fig. 1. Injection locking of various laser structures: (a) VCSEL, (b) Fabry–Perot, (c) DFB.

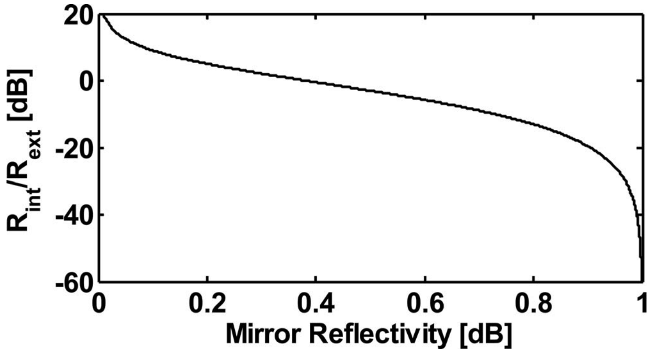

Fig. 2. Ratio of internal and external injection ratios for different mirror reflectivities.

ratio, which is easily determinable by empirical methods. Note also that Eq. (4) is usable for reflection-type injection locking experiments and can be modified for transmission-type OIL. Equation (4) is plotted versus mirror power reflectivities in Fig. 2, which shows that for extremely high mirror reflectivities (i.e., VCSELs), much of the incident light does not transmit into the cavity. For a typical EEL with air facets ($r \approx 0.3$), this ratio is close to unity. However, for a VCSEL with $r=0.99$, the value is $\approx 10^{-4}$.

The coupling coefficient, κ, for injection-locked lasers has been shown to be important for determining the efficiency of the injection process. It is typically defined as [7]

$$\kappa = \frac{1}{\tau_{rt}} = \frac{v_g}{2L}, \tag{5}$$

where τ_{rt} is the cavity round-trip time, v_g is the cavity group velocity, and L is the cavity length. Physically, it means that the injected light must distribute itself across the entire laser cavity. Therefore, longer cavities have poorer injection efficiencies; the longer cavity dilutes the injection's effects. A VCSEL with $L=2\ \mu$m versus an EEL with $L=500\ \mu$m would benefit from a κ that is 250 times larger than its EEL counterpart.

The resonance frequency enhancement is [5,6]

$$\Delta\omega_R = -\kappa\sqrt{R_{int}}\sin\phi_0, \tag{6}$$

where ϕ_0 is the steady-state phase difference between master and slave laser fields. The resonance frequency enhancement reaches its maximum when $\phi_0=-\pi/2$, which occurs at the positive detuning edge of the stable locking range [7]:

$$-\kappa\sqrt{R_{int}}\sqrt{1+\alpha^2} < \Delta\omega < \kappa\sqrt{R_{int}}. \tag{7}$$

Using Eqs. (5) and (6), and the upper bound of Eq. (7), we obtain the maximum resonance frequency enhancement for a given injection ratio:

$$\Delta\omega_{R,max} = \kappa\sqrt{R_{int}} = \frac{v_g}{2L}\sqrt{R_{int}}. \tag{8}$$

This equation is dependent only on the cavity round-trip time and the injection ratio. Equation (8) suggests that high resonance frequency enhancement would favor short cavity lasers. However, one must remember that short cavity lasers require high reflectivity mirrors, which reduces the internal injection ratio as shown in Fig. 2. To find the trade-off between cavity length and mirror reflectivity, we use Eq. (4) to relate Eq. (8) to the external injection ratio:

$$\Delta\omega_{R,max} = \frac{v_g}{2L}\cdot\frac{1-r}{\sqrt{r}}\sqrt{R_{ext}}. \tag{9}$$

Note that the quality factor of a loss-free Fabry–Perot cavity (coupling Q) with mirror reflectivities of r and a cavity length of L is [8]

$$Q \equiv \frac{\omega_0}{\omega_{1/2}} = \frac{\omega_0 L}{v_g}\cdot\frac{\sqrt{r}}{1-r}, \tag{10}$$

where ω_0 is the laser frequency and $\omega_{1/2}$ is the full width at half-maximum bandwidth of the cavity resonance; $\Delta\omega_{R,max}$ can be simplified to

$$\Delta\omega_{R,max} = \frac{\omega_0}{2Q}\sqrt{R_{ext}}. \tag{11}$$

This equation removes the dependency of the resonance frequency enhancement from r and L and relates it to a single cavity parameter, Q. It also states that to obtain a high resonance frequency, we should design a laser cavity with a low Q. Interestingly, a typical EEL with $L=500\ \mu$m and $r=0.3$ has the same Q ($=6.7\times 10^3$) as a typical VCSEL with $L=2\ \mu$m and $r=0.995$. Equation (11) states that both lasers would have the same maximum resonance frequency enhancement. The VCSEL's high coupling coefficient, κ, is compensated by the decreased ratio of light transmitted into the cavity. To increase the maximum resonance frequency, lasers with lower Q should be designed. Figure 3 shows the calculated frequency responses of two injection-locked lasers with different Q. For a given injection ratio, the laser with a lower Q [Fig. 3(b)] will attain a higher resonance frequency. One must keep in mind that, ultimately, the practical limits of resonance frequency enhancement may re-

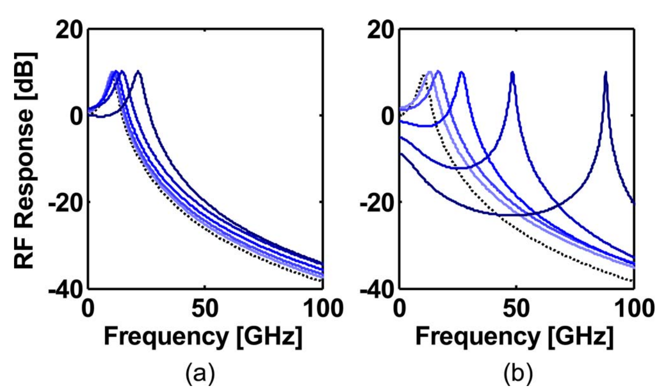

Fig. 3. (Color online) Frequency responses for two different lasers: (a) $Q=13{,}320$, (b) $Q=3330$. The curves in each set are the maximum resonance frequency at a given injection ratio. Light to dark curves correspond to $R_{ext}=-10$, -5, 0, 5, and 10 dB, respectively. Dotted curves correspond to the free-running response.

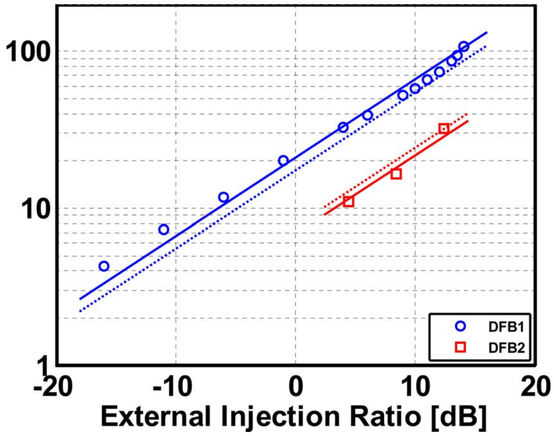

Fig. 4. (Color online) Comparison of theory with experimental data for maximum resonance frequency enhancement. The points are sets of $\Delta\omega_{R,max}$ at different injection ratios, for two different lasers. Solid lines, fits of Eq. (11); dotted lines, calculated $\Delta\omega_{R,max}$ based on DFB stop-band width.

sult from other effects, i.e., when the increased detuning causes locking to prefer the next cavity mode. This may be farther away for VCSELs than distributed-feedback (DFB) lasers, owing to the VCSEL's shorter effective cavity length.

To validate the theory, we fit Eq. (11) to our experimental data (Fig. 4) by using DFB lasers with a $\kappa_{grating}L$ product of ~4. We maximized the resonance frequency at each injection ratio value. It should be noted that $\Delta\omega_{R,max}$ is calculated from the free-running resonance frequency, ω_{R0}, and the enhanced resonance frequency, ω_R. The two can be related through [6,9]

$$\omega_R^2 = \omega_{R0}^2 + \Delta\omega_R^2. \quad (12)$$

This is a more accurate prediction than previous literature [5]. It is derived from linearized rate equations [9] and better models the fact that the locked laser returns to its free-running resonance frequency under negative detuning or low-injection regimes. For each laser, the $\Delta\omega_{R,max}$ versus R_{ext} curve fits with a line of log–log slope 1/2, agreeing well with the prediction by Eq. (11). DFB1 fits well over a span of 30 dB. The extracted Q values of DFB1 and DFB2 are 4660 and 14,100, respectively. From the optical spectrum, the stop-band widths of the two DFBs were estimated to be 1 and 2 nm, respectively. From this, the calculated Q values, assuming no facet reflectivity, are 5570 and 12,600, respectively [8]. The maximum resonance frequency enhancement based on the calculated Qs is plotted in Fig. 4, showing good agreement and the general trend that lower Q factor results in higher maximum resonance frequencies.

Using Eqs. (10) and (11) and the relationship between $\omega_{1/2}$ and photon lifetime (τ_c) of the cavity mirrors, $\omega_{1/2}=1/\tau_c$, we obtain

$$\tau_c \cdot \Delta\omega_{R,\max} = \frac{1}{2}\sqrt{R_{\text{ext}}}. \quad (13)$$

Equation (13) can be seen as a time–bandwidth product, which increases with higher injection ratio. This defines a trade-off between high resonance frequency and low threshold currents.

We can also use Eq. (11) to delineate the upper edge of the locking range. Interestingly, Adler [10] and similarly Slater [11] cite an identical boundary on the locking range of injection-locked electronic oscillators:

$$\Delta\omega = \frac{\omega_0}{2Q}\sqrt{\frac{P_i}{P_0}}. \quad (14)$$

The similarity between electronic and optical oscillator theory suggests that the theory is universal to all types of driven nonlinear oscillators, including different optical cavity designs, and that cavity Q is the main factor affecting the resonance frequency for a given injection ratio.

In summary, we have derived a universal formula for the maximum resonance frequency enhancement of an injection-locked semiconductor laser in terms of the quality factor (Q) and the external injection ratio. The enhancement increases with the square root of the external injection ratio but decreases with Q. With this model, we find that typical lasers of different lengths have comparable performance for the same external injection ratio, provided they have similar Q. Finally, we show that the time–bandwidth product of injection-locked lasers is equal to one half of the square root of the external injection ratio. The results presented here clearly identify the design trade-off between the threshold of the laser and the maximum resonance frequency enhancement, and they can serve as a universal guideline to optimize the performance of injection-locked lasers.

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