Phase Noise Spectrum and Carrier Power Modeling of High Performance Optomechanical Oscillators

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Abstract: We present a model for phase noise that fits experimentally observed trends in our high performance optomechanical oscillators (OMO) demonstrating a phase noise of -102dBc/Hz at 1kHz offset from a 74MHz carrier.
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1. Introduction

Radiation Pressure driven OptoMechanical Oscillators (RP-OMO's) have shown great promise as monolithic RF frequency sources with an optical carrier [1,2]. In such devices, the dynamic interaction of photon pressure and mechanical position generates a parametric gain sending the mechanical resonator into self-sustained oscillation. A critical figure of merit in any frequency reference is the phase noise spectrum, a measure of frequency stability. Thus, in order to design OMO's properly, a model for the phase noise spectrum is necessary. Previous models of RP-OMOs [3] assume a Lorentzian noise spectrum which isn't always the case, especially in high optical Q cavities [4]. Additionally, we model the carrier power and phase noise separately which elucidates the negative effect of high optical Q on phase noise, a previously observed phenomenon [2,5]. Model results are compared with experiments when important parameters such as mechanical quality factor (Q_m) and optical quality factor (Q_o) are varied.

This work focuses on optical whispering gallery mode ring RP-OMO's, cf. Fig1a. In such a device, the optical and mechanical modes are collocated and the device periphery is displaced radially outward from the center. Minimization of mechanical anchor loss is achieved with thin spokes to support the optomechanical rings [6]. Two types of RP-OMO materials are investigated: nitride with Q_m as high as10,000[1] and Q_o of 110,000 and PhosphoSilicate Glass (PSG) with a Q_o as high as 5.3 million and Q_m of 7,200.

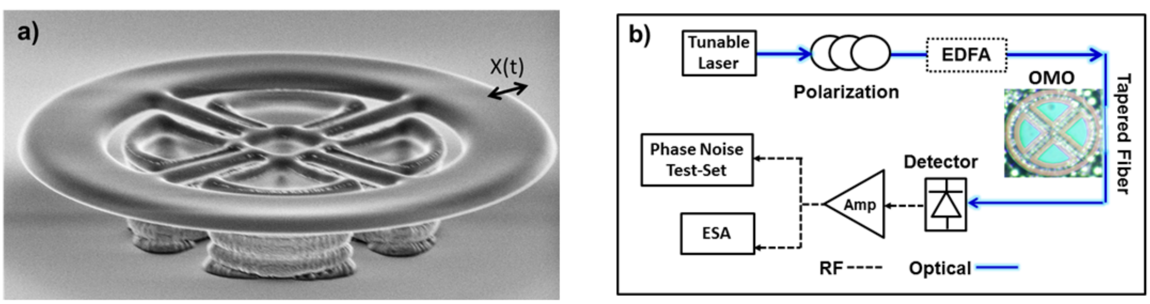

Fig. 1 (a) SEM of PSG OMO device with radial displacement, *x(t)* (b) Measurement setup. Light is amplified by an EDFA (optional) and evanescently coupled into the cavity by tapered fiber. The light modulation is detected and analyzed electronically.

2. The Noise Spectrum

Noise sources present in the OMO produce jitter in an otherwise pure sinusoidal output. Thus, instead of a single frequency tone, the RF power is spread across frequencies near the carrier. The phase noise spectrum is defined:

$$\mathcal{L}(f') = 10 \cdot \log_{10}\left(\Delta f^{-1} \cdot P_{phase}(f')/P_{osc}\right)$$

where P_{phase} is the noise in the detected output as a function of offset frequency f' from the carrier. P_{osc} is the oscillator carrier power and Δf is the noise bandwidth.

The noise in optical power at the cavity output in frequency space, $\delta P_{out}(\Omega)$, is found from a small signal model [4,7] . $\delta P_{out}(\Omega)$ depends on other noises including the field noise at the cavity input, and noise in the radial displacement assumed to be dominated by thermal Brownian motion. For a thermally limited oscillator, P_{phase} is half the RF spectral noise power after photo-detection which is proportional to $|\delta P_{out}|^2$.

3. Oscillator Carrier Power

To find the carrier power, we extend the result of [8] which assumes sinusoidal displacement to arrive at the output optical power in time. Specifically, we only keep terms oscillating at the mechanical angular resonant frequency Ω_m which after manipulation gives, $P_{out}(t)|_{\Omega_m} = \hbar\omega_o(A(t) + B(t))$ where,

$$A(t) = -2\frac{|s_{in}(t)|^2}{\tau_{ex}}\sum_n \frac{J_n(\beta)}{(\kappa/2)^2+(\bar{\Delta}+n\Omega_m)^2}\left[(\bar{\Delta}+n\Omega_m)cos\Omega_m t\big(J_{n+1}(\beta)-J_{n-1}(\beta)\big)+\frac{\kappa}{2}sin\Omega_m t\big(J_{n+1}(\beta)+J_{n-1}(\beta)\big)\right]$$

$$B(t) = \frac{|s_{in}(t)|^2}{\tau_{ex}^2}\sum_n \frac{J_n(\beta)}{(\kappa/2)^2+(\bar{\Delta}+n\Omega_m)^2}\Bigg[-\Omega_m\frac{\kappa}{2}\cos\Omega_m t\left(\frac{J_{n+1}(\beta)}{(\kappa/2)^2+(\bar{\Delta}+(n+1)\Omega_m)^2}+\frac{J_{n-1}(\beta)}{(\kappa/2)^2+(\bar{\Delta}+(n-1)\Omega_m)^2}\right)+ sin\Omega_m t\left(J_{n+1}(\beta)+J_{n-1}(\beta)-\frac{\Omega_m J_{n+1}(\beta)((\bar{\Delta}+(n+1)\Omega_m)}{(\kappa/2)^2+(\bar{\Delta}+(n+1)\Omega_m)^2}+\frac{\Omega_m J_{n-1}(\beta)((\bar{\Delta}+(n-1)\Omega_m)}{(\kappa/2)^2+(\bar{\Delta}+(n-1)\Omega_m)^2}\right)\Bigg] \quad (2)$$

with input field $s_{in,}$ optical coupling time constant τ_{ex}, loaded cavity bandwidth κ, laser detuning $\bar{\Delta}$, modulation index $\beta \equiv -x_o \cdot g_o/\Omega_m$ displacement amplitude x_o and optomechanical coupling g_o. The output field consists of sine and cosine components each weighted by the optical cavity Lorentzian lineshape $((\kappa/2)^2 + (\bar{\Delta}+\Omega_m)^2)^{-1}$ evaluated at integer multiples of Ω_m. Thus, only terms in which $n\Omega_m<\kappa/2$ contribute to each sum.

4. Experimental Verification

Several experiments have been carried out to validate our model [1,5]. In each experiment the laser detuning, and external waveguide coupling were optimized for low phase noise. In the most recent experiment, Q_m was enhanced by placing a high performance silicon nitride OMO in vacuum to reduce gas damping resulting in an increase in Q_m from 1,800 (air) to 10,400 (vacuum). The improved Q_m yielded a phase noise of -102dBc/Hz at 1KHz offset with 74MHz carrier – the best phase noise for an RP-OMO thus far. Our model accurately predicts the observed behavior with fit parameters being the input noise proportional to $\sqrt{P_{in}}$, fiber coupling τ_{ex}, and detuning.

In another experiment, three optical resonances with Q_o ranging from 300,000 to 5.3 million in the same device were excited at atmosphere[5]. Here, the laser power was held at twice the threshold power. Both data and modeling show that an increased optical Q has an adverse effect on phase noise. Our modeling indicates that this is due to reduced carrier power in the high Q_o case. Physically, a high optical Q_o device is efficient at filtering out higher order RF frequency harmonics produced by the non-linear interaction of the circulating photons with the sinusoidally varying cavity radius. This filtering however, negates the possibility of photons at the n^{th} harmonic feeding energy back to the fundamental carrier.

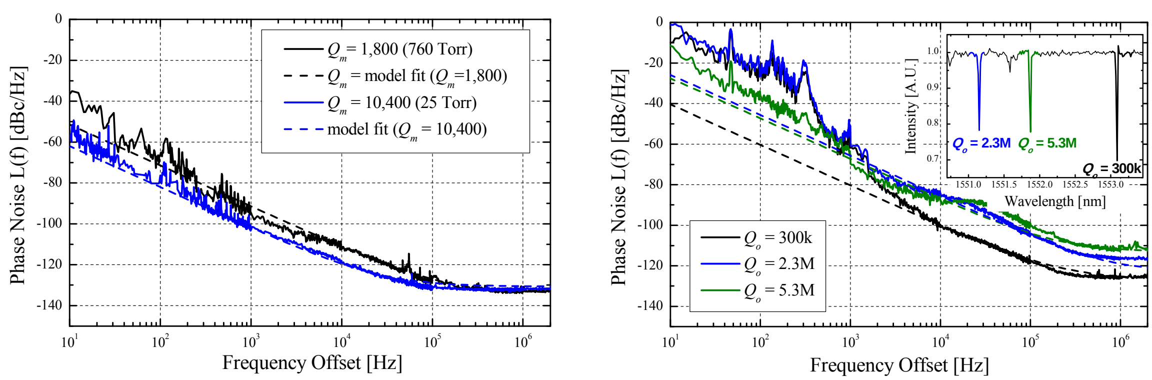

Fig. 2 (a) Fitted and measured phase noise of 25µm outer radius nitride OMO with f_m=74MHz (b) Measured and model fit (dashed) phase noise for three optical resonances in the same 50µm PSG device. Inset: three optical resonances seen in transmission through tapered fiber.

4. References

[1] T.O. Rocheleau, A.J. Grine, K.E. Grutter, R.A. Schneider, N. Quack, M.C. Wu, C.-T. Nguyen, *2013 IEEE MEMS*, Taipei, Taiwan, 20-24 Jan.
[2] S. Tallur, S. Sridaran, and S. Bhave, *2012 IEEE MEMS*, Paris, France, 29Jan.-2 Feb. 2012.
[3] S. Tallur, S. Sridaran, S.A. Bhave, T. Carmon, *IEEE FCS* 2010, pp. 268-272.
[4] A. Schliesser *et al.*, *New Journ. of Physics*, vol. 10, no. 9, 2008.
[5] A. Grine, N. Quack, K. Grutter, T. Rocheleau, *et al.*, *Proceedings, IEEE Int. Conf. on Opt. MEMS and Nanophotonics*, 2012, pp. 51-52.
[6] S.-S. Li *et al.*, *IEEE MEMS* 2004, pp. 821-824
[7] C. Fabre *et al.*, *Phys. Rev. A*, vol. 49, no. 2, p. 1337, 1993
[8] A. Schliesser et. al., *Nature Physics,* vol. 4, pp. 415-419, 2008