# Circuit Theory of Optical Antenna Shedding Light on Fundamental Limit of Rate Enhancement

**Michael Eggleston, Kevin Messer, Eli Yablonovitch, and Ming C. Wu**
*Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720*
*eggles@berkeley.edu*

**Abstract:** A circuit model of a single-element linear optical antenna is presented. It agrees well with FDTD simulations and predicts spreading resistance will ultimately limit the maximum rate enhancement an efficient antenna can achieve to ~10,000.
**OCIS codes:** (350.4238) Nanophotonics and photonic crystals; (250.5403) Plasmonics,

## 1. Introduction

Optical antennas have been demonstrated to enhance the rate of spontaneous emission from dye molecules[1] and semiconductor materials[2], enhance absorption[1,3] in these same materials, and significantly increase Raman scattering[4]. The most common type of optical antenna is the center-fed dipole antenna, consisting of a bar of metal with a gap in the middle where the emitting or absorbing material is placed. Modeling of these structures is typically performed by numerical simulators such as FDTD or FEM. There has yet to be a simple closed-form theoretical model that can predict an antennas performance based on its geometry alone. Because of this, there is little understanding of what the maximum rate enhancement an optical antenna can achieve.

In this paper we propose a simple circuit model for the optical dipole antenna. By using an antenna's physical dimensions, its resonant frequency, radiation efficiency, and radiated power can be calculated without the need for complex computer simulations. The model is applicable to any single element linear antenna (rod, bowtie, etc.), and several antenna structures are modeled and shown to agree well with results from a full-wave FDTD simulation. The model predicts a sharp decrease in efficiency at small gaps caused by current crowding near the excitation dipole regardless of antenna structure. This ultimately limits the maximum rate enhancement an efficient antenna can achieve.

## 2. Circuit Model

The simplest dipole antenna can be modeled as a metal cylinder of radius, r, and a small gap, d, at the center. This metal rod will have distributed inductance and capacitance as well as the discrete capacitance that the gap introduces. These reactances will determine the resonant frequency of the antenna. As shown in Fig. 1.a, by breaking the distributed capacitance into an "inner" and "outer" capacitance, a simplified discrete model can be constructed that accurately predicts the antenna's resonant frequency.

An antenna will also have several loss mechanisms, mainly ohmic resistance from losses in the bulk metal antenna, radiation resistance from radiation losses, and spreading resistance (Fig. 1.b). Spreading resistance is introduced when the antenna is excited by an oscillating dipole from the emitter material in the gap. As the gap shrinks down, the dipole fields are increasingly confined to a small region on the metal surface in the gap. This field induces current that spreads out to the full width of the antenna, but the initial current crowding introduces significant loss. This loss is intensified by the anomalous skin effect[5] which effectively increases the resistivity of the metal near the excitation source. As shown in Fig. 1.c, the radiation losses are dominant in a well-designed antenna until the gap reaches sub 5nm, at which point spreading resistance takes over as the dominant loss mechanism.

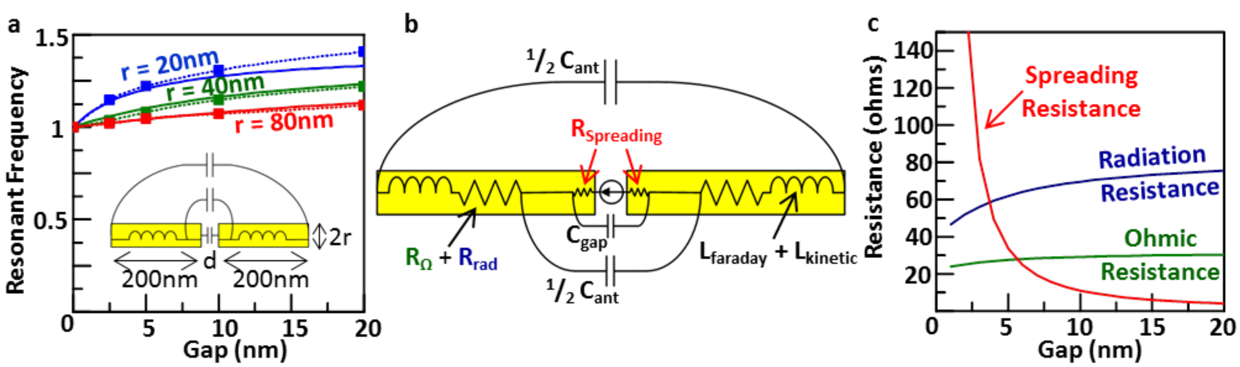

**Figure 1**. (a) Normalized antenna resonant frequency as a function of the gap spacing for an antenna with two 200nm long arms and various radii. Solid lines are predicted by the inset model, solid boxes connected by a dashed line show the frequency simulated with FDTD. (b) Circuit model of an optical antenna superimposed on a simple dipole antenna schematic. (c) Resistances in an optical antenna resonant at 1500nm with a radius of 20nm for various gap spacings.

## 3. Results

An important use of optical antennas is increasing the rate of spontaneous emission from a nanoscale emitter. The circuit model in Fig. 1.b was used to calculated the efficiency and rate enhancement of such an antenna for an oscillating dipole emitting at 200THz. The rate enhancement is the ratio of radiated power with the antenna to that of the dipole alone in free space. Fig. 2.a shows the enhancement and efficiency predicted by the model as well as the values calculated with FDTD for a dipole antenna with a radius of 20nm and vaccuum occupying the gap. The predicted values agree very well with simulation. An additional plot of efficiency that includes the anomalous skin effect (not present in simulation) shows only a moderate decrease in efficiency until the gap is reduced to sub 5nm at which the efficiency drops off very quickly.

A practial optical antenna, however, will not have vaccuum in the gap. Fig. 2.b shows the same antenna structure as Fig. 2.a but with a high dielectric in the gap similar to a typical III/V semiconductor (n=3.4).The high index gap increases the gap capacitance significantly, creating a shunt path for the current induced by the radiation dipole. This reduces the enhancement by orders of magnitude and since shunt current still suffers from spreading resistance, the efficiency drops off at much larger gaps.

To remedy the large shunt capacitance created with a high index gap, two other antenna geometries were modeled (Fig. 2.c). The first structure has rounded tips at the gap. Similar to a tapered gap or bowtie antenna, this creates a much smaller effective gap area and significantly reduces the gap capacitance. The result is a restoration of the large enhancements seen in Fig. 2.a. A second structure, the arch-antenna[2], creates an inductive loop over the emitter material. If correctly tuned, this inductance can effectively reduce the gap capacitance, nearly eliminating shunting current and achieving even higher rate enhancement. For all cases studied, however, the antenna efficiency drops off quickly below a 5nm gap. This caps the maximum rate enhancement possible with an optical antenna to ~10,000 for an efficient antenna regardless of dimension.

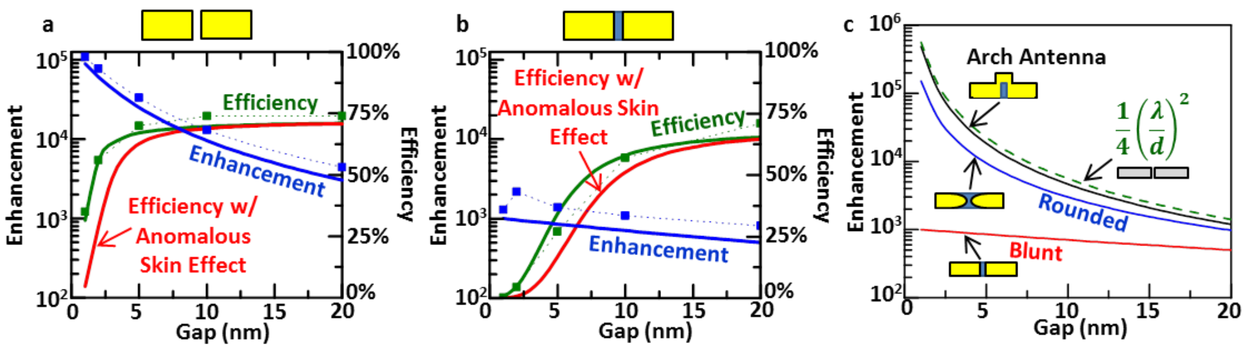


**Figure 2.** Enhancement and efficiency of (a) dipole antenna with vaccuum gap and (b) high index gap. Solid lines are values predicted by the circuit model while solid squares are values calculated with FDTD. (c) Comparison of rate enhancement of several different antenna geometries.

In conclusion, a simple circuit model for single-element linear antenna was proposed comprised entirely of elements based off the antennas physical parameters. The circuit predicts antenna efficiency and rate enhancement of a dipole emitter in agreement with values from full-wave FDTD simulations. For an efficient device, spreading resistance limits the maximum rate enhancement of an optical antenna to ~10,000 for a well-designed structure.

## 4. References


[1] A. Kinkhabwala, Z. Yu, S. Fan, Y. Avlasevich, K. Müllen, and W. E. Moerner, "Large single-molecule fluorescence enhancements produced by a bowtie nanoantenna," *Nature Photon*, Oct. 2009.

[2] M. Eggleston, N. Kumar, K. Messer, L. Zhang, E. Yablonovitch, and M. C. Wu, "Efficient Rate Enhancement of Spontaneous Emission in a Semiconductor nanoLED," presented at the Frontiers in Optics Conference, 2012.

[3] D. A. B. Miller, "Nanomettalic Concentration for Enhanced Detection," IEEE Photonics 2011.

[4] T. J. Seok, A. Jamshidi, M. Eggleston, and M. C. Wu, "Mass-producible and efficient optical antennas with CMOS-fabricated nanometer-scale gap," *Opt. Express*, vol. 21, no. 14, pp. 16561–16569, Jul. 2013.

[5] P. W. Gilberd, "The anomalous skin effect and the optical properties of metals," *Journal of Physics F: Metal Physics*, vol. 12, no. 8, p. 1845, 1982.